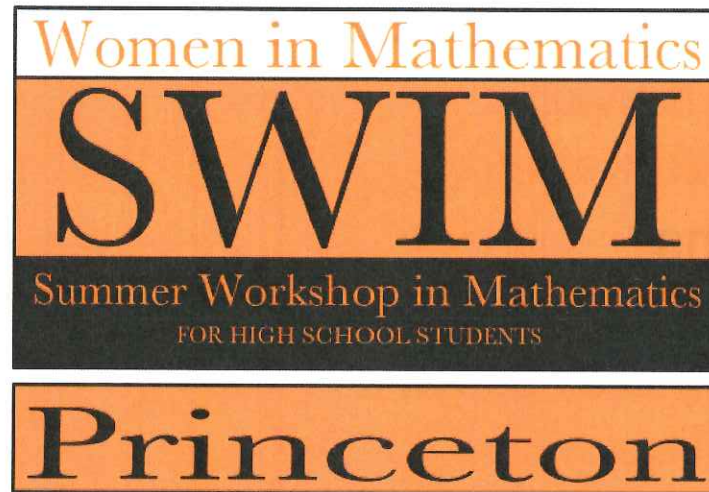


Introduction to Abstract Algebra

with Applications to Social Systems



Course II
Lecture
Notes
1 of 7

Princeton SWIM 2010
Instructor: Taniecea A. Arceneaux
Teaching Assistants: Sarah Trebat-Leder and Amy Zhou

Course Outline

- Introduction and Matrix Algebra Review
- Communication and Sociometric Matrices
 - Directed Networks
 - Undirected Networks
- Markov Chains
 - Transition Probabilities
 - Stationary Distribution
 - The DeGroot Model
- Equivalence Classes
- Marriage Rules in Primitive Societies
 - Family Tree Diagrams
 - *Kariera* Society Marriage Structure
 - Introduction to Group Theory
- Additional Topics

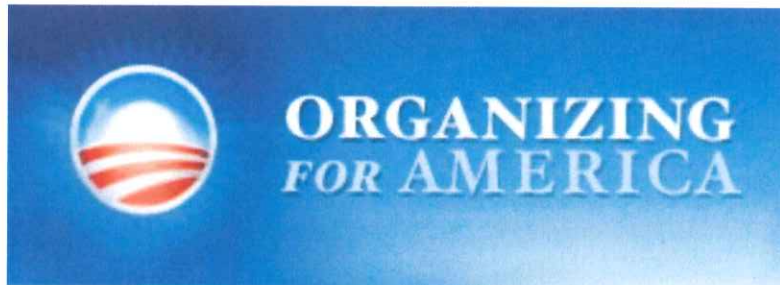
Introduction

Recent Social Networks of Interest

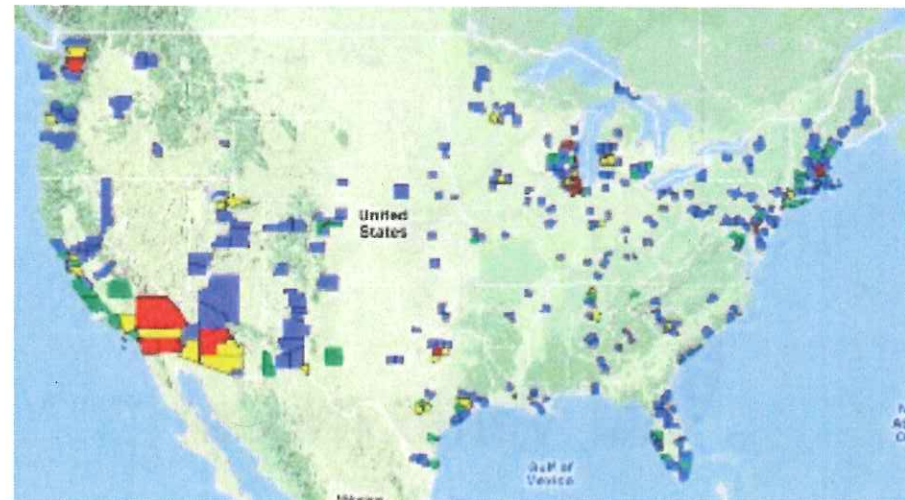
facebook

 Blogger

twitter



Barack Obama
Presidential Campaign

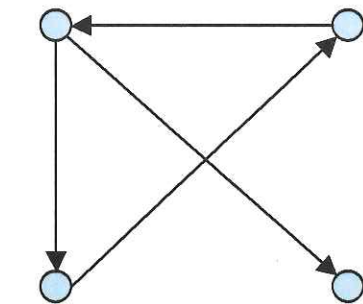


Tracking H1N1

Sociometric Matrices

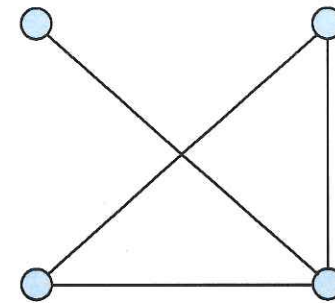
Social Networks as Graphs

Directed Graph



 Vertex (Node)

Undirected Graph

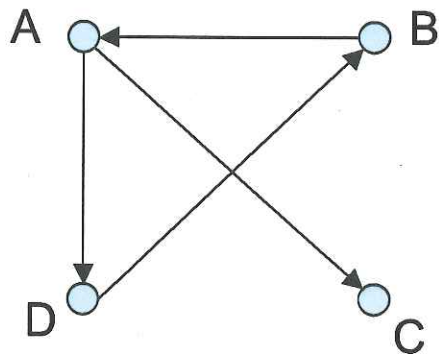


Edge (Link) 

Sociometric Matrices

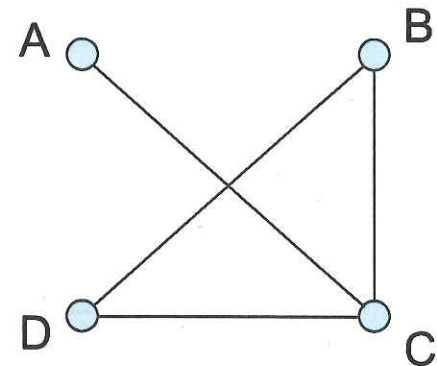
Social Networks as Graphs

Directed Graph



$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Undirected Graph

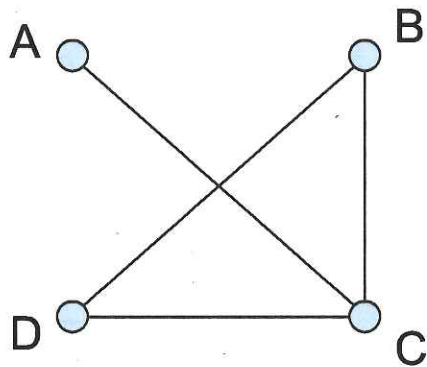


$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Sociometric Matrices

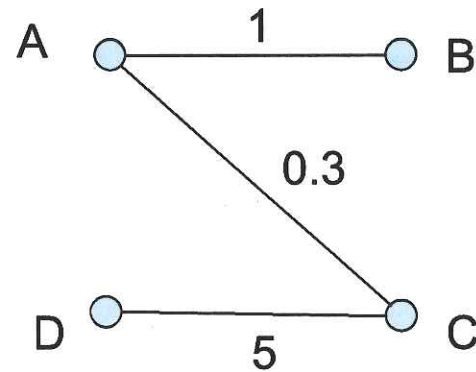
Social Networks as Graphs

Unweighted Graph



$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Weighted Graph



$$\begin{pmatrix} 0 & 1 & 0.3 & 0 \\ 1 & 0 & 0 & 0 \\ 0.3 & 1 & 0 & 5 \\ 0 & 0 & 5 & 0 \end{pmatrix}$$

The Rise of the Medici

Padgett and Ansell (1993)

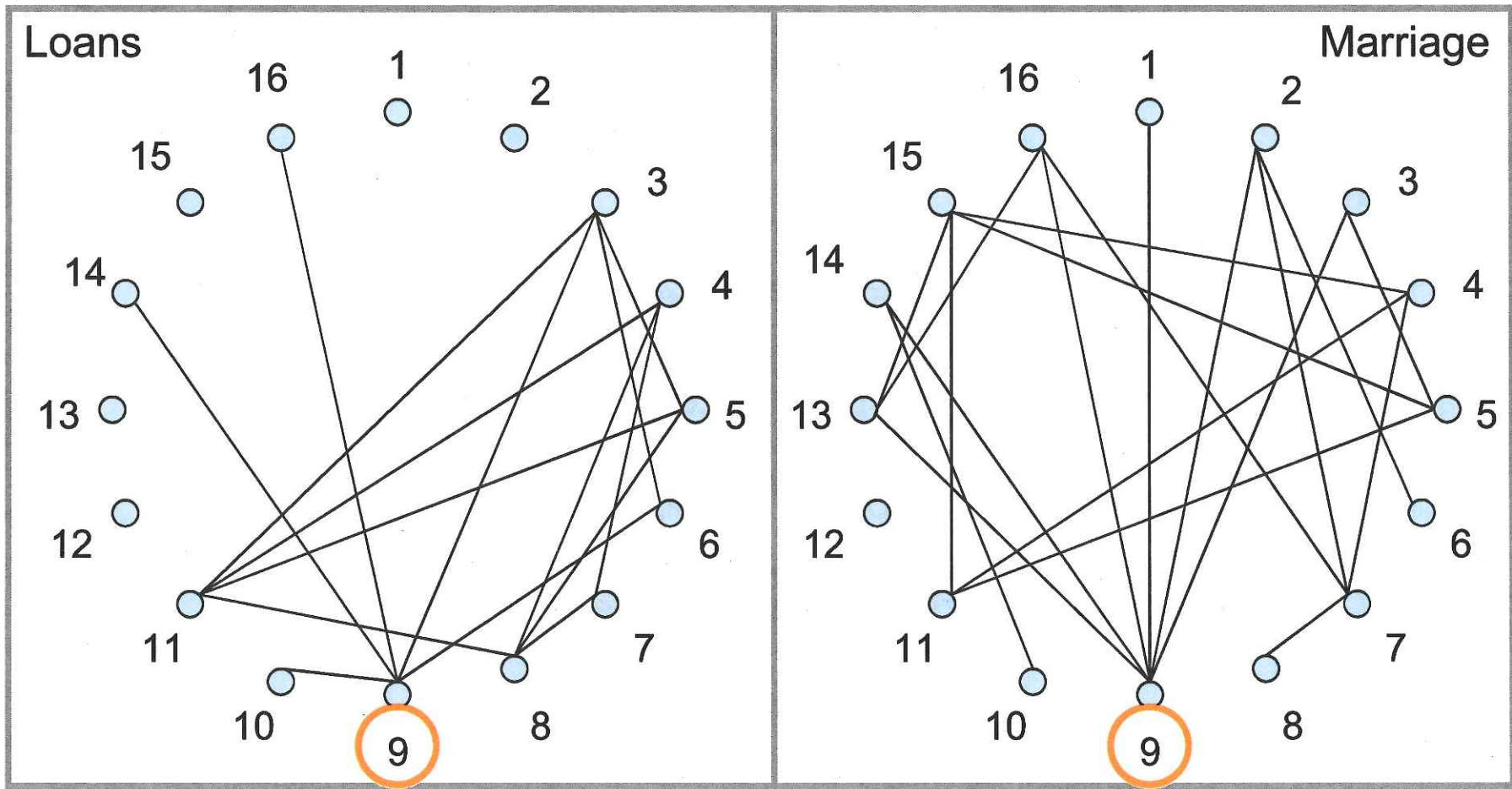
Padgett, John F. and Christopher K. Ansell. 1993. "Robust Action and the Rise of the Medici, 1400-1434," *The American Journal of Sociology*, Vol. 98, No. 6, pp. 1259-1319.

Background:

- Italian Renaissance began in 15th century Florence, Italy.
- The cultural movement impacted European intellectual life.
- Affected literature, philosophy, art, politics, science, and religion.
- Characterized by the rise in power of the Medici family.

The Rise of the Medici

Business (Loans) and Marriage Relations



Markov Chains

The DeGroot Model

Consider a group of n individuals

- Original opinions - Vector of Probabilities

$$p(0) = (p_1(0), p_2(0), \dots, p_n(0))$$

- Weight placed on others' opinions - Transition Probability Matrix

$$T_{ij} = \begin{pmatrix} t_{1,1} & t_{1,2} & \cdots & t_{1,n} \\ t_{2,1} & t_{2,2} & \cdots & t_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n,1} & t_{n,2} & \cdots & t_{n,n} \end{pmatrix}$$

Row Stochastic

- Updated opinions - Markov Chain

$$p(t) = Tp(t-1) = T^t p(0)$$

The DeGroot Model

Objectives

- To describe how a group of individuals can reach a consensus of beliefs
- To pool the opinions of each individual into a single group consensus
- To form a common probability distribution of the group consensus
- To determine how *central* an individual is in the network

DeGroot, Morris H. 1974. "Reaching a Consensus." *Journal of the American Statistical Association*, Vol. 69, No. 345, pp. 118-121.

The DeGroot Model

Implications

- Each new opinion depends only on opinion of previous time period
- Convergence - T is strongly connected (irreducible) and aperiodic

$$\pi T = \pi \quad \text{Stationary Distribution}$$

- No Convergence - States of the chain can form at least 2 disjoint closed sets

Primitive Societies

Marriage Rules

- Each individual has a marriage type.
- Only individuals of the same marriage type may marry.
- Marriage type is determined by parents' type and gender.
- Brother-sister marriages are not allowed.
- Some first-cousin marriages are allowed.

Family Tree Diagrams

First Generation

Legend:



Male



Female



Marriage

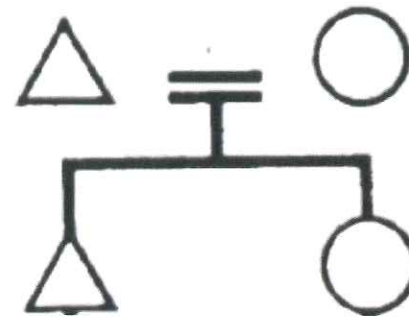


Descendent



Sibling

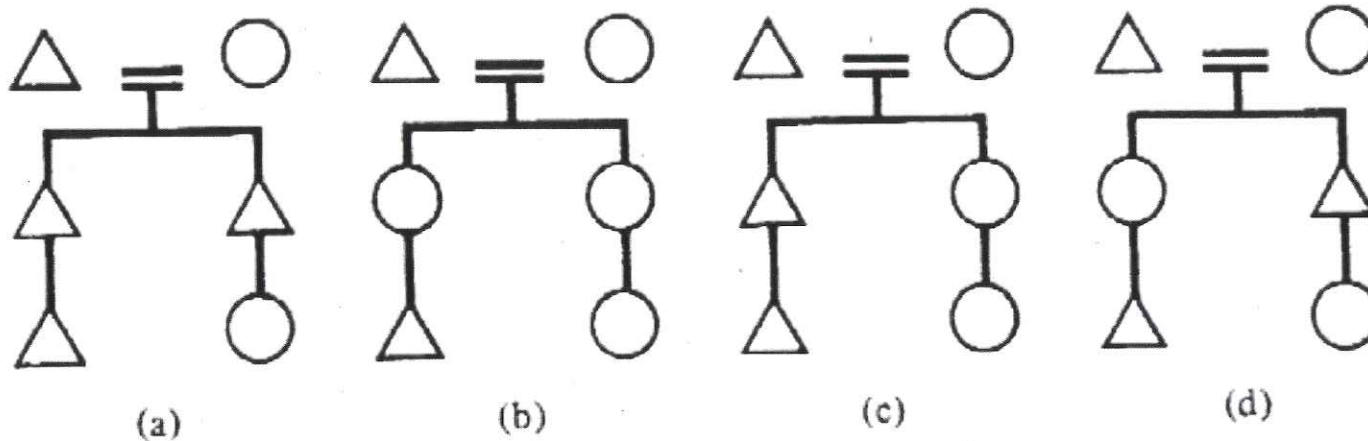
Brother-sister relationship



Family Tree Diagrams

Second Generation

Four types of first-cousin relationships

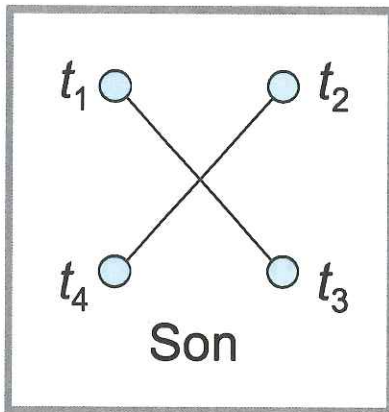


Primitive Marriage Rules

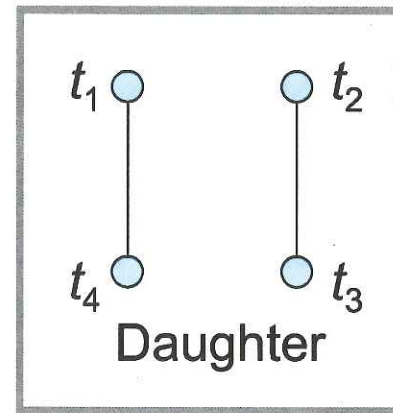
Example - *Kariera* Society, First Generation

Table 1. *Kariera* Marriage Types – First Generation

| | | | | |
|----------|-------|-------|-------|-------|
| Parents | t_1 | t_2 | t_3 | t_4 |
| Son | t_3 | t_4 | t_1 | t_2 |
| Daughter | t_4 | t_3 | t_2 | t_1 |



$$S = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$



$$D = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

NOTE: S and D are permutation matrices

Additional Topics

Southern Women Data

Davis, Allison, Burleigh B. Gardner, Mary R. Gardner. 1941. *Deep South: A Social Anthropological Study of Caste and Class*. Chicago: The University of Chicago Press.

Goal:

- Examine relation between social class and informal interaction

Data Collection:

- Spent 9 months in Natchez, Mississippi
- Observed 18 women during 14 informal social events (“a day’s work behind the counter of a store, a meeting of a women’s club, a church supper, a card party, a supper party, a meeting of the PTA, etc”)
- Recorded participation using “interviews, the records of participant observers, guest lists, and the newspapers”

Southern Women Data

Davis, Gardner, and Gardner (1941)

| Name of Participant or Group I | Code Numbers and Dates of Social Events Reported by Old City Field | | | | | | | | | | | | |
|----------------------------------|--|-----|------|------|------|-----|------|------|------|-----|------|------|------|
| | 4/27 | 5/2 | 5/12 | 5/26 | 5/31 | 6/6 | 6/13 | 6/20 | 6/27 | 7/4 | 7/11 | 7/18 | 7/25 |
| 1. Mrs. Evelyn Jefferson | X | X | X | X | X | X | | X | X | X | | | |
| 2. Miss Laura Manderline | X | X | X | X | X | X | | X | X | X | | | |
| 3. Miss Theresa Anderson | | X | X | X | X | X | | X | X | X | | | |
| 4. Miss Brenda Rogers | X | | X | X | X | X | | X | X | X | | | |
| 5. Miss Charlotte McDowd | | | X | X | X | X | | X | X | X | | | |
| 6. Miss Frances Anderson | | | X | | X | X | | X | X | X | | | |
| 7. Miss Eleanor Nye | | | | | X | X | | X | X | X | | | |
| 8. Miss Pearl Ogleshorpe | | | | | X | X | | X | X | X | | | |
| 9. Miss Ruth DeSand | | | | | X | X | | X | X | X | | | |
| 10. Miss Verne Sanderson | | | | | | | | X | X | X | | | |
| 11. Miss Myra Liddell | | | | | | | | X | X | X | | X | |
| 12. Miss Katherine Rogers | | | | | | | | X | X | X | | X | X |
| 13. Mrs. Sylvia Avondale | | | | | | | | X | X | X | | X | X |
| 14. Mrs. Nora Fayette | | | | | | X | | X | X | X | | X | X |
| 15. Mrs. Helen Lloyd | | | | | | | | X | X | X | | X | |
| 16. Mrs. Dorothy Murchison | | | | | | | | X | X | X | | | |
| 17. Mrs. Olivia Carleton | | | | | | | | X | X | X | | | |
| 18. Mrs. Flora Price | | | | | | | | X | X | X | | | |

Southern Women

Research Questions

- Is the network of Southern women connected through social events?
- Do distinct social groups exist among these Southern women?
- Which of the women are more highly connected than others?

Southern Women

Woman by Event Matrix

$$S =$$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|-----------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| Eleanor | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| Brenda | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| Dorothy | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Verne | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Flora | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Olivia | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Laura | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| Evelyn | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| Pearl | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| Ruth | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Sylvia | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| Katherine | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| Myra | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Theresa | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| Charlotte | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Frances | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| Helen | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Nora | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

Southern Women

Weighted Matrix - Woman by Woman

$$A'_{ij} = SS^T = \begin{pmatrix} 4 & 4 & 1 & 2 & 0 & 0 & 4 & 3 & 2 & 3 & 2 & 1 & 1 & 4 & 2 & 3 & 2 & 2 \\ 4 & 7 & 1 & 2 & 0 & 0 & 6 & 6 & 2 & 3 & 2 & 1 & 1 & 6 & 4 & 4 & 2 & 2 \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 \\ 2 & 2 & 2 & 4 & 1 & 1 & 2 & 2 & 2 & 3 & 4 & 3 & 3 & 3 & 1 & 1 & 3 & 3 \\ 0 & 0 & 1 & 1 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 2 \\ 4 & 6 & 1 & 2 & 0 & 0 & 7 & 6 & 2 & 3 & 2 & 1 & 1 & 6 & 3 & 4 & 2 & 2 \\ 3 & 6 & 2 & 2 & 1 & 1 & 6 & 8 & 3 & 3 & 2 & 2 & 2 & 7 & 3 & 4 & 1 & 2 \\ 2 & 2 & 2 & 2 & 1 & 1 & 2 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 0 & 2 & 1 & 2 \\ 3 & 3 & 2 & 3 & 1 & 1 & 3 & 3 & 2 & 4 & 3 & 2 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 4 & 1 & 1 & 2 & 2 & 2 & 3 & 7 & 6 & 4 & 3 & 1 & 1 & 4 & 6 \\ 1 & 1 & 2 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 6 & 6 & 4 & 2 & 0 & 1 & 3 & 5 \\ 1 & 1 & 2 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 4 & 4 & 4 & 2 & 0 & 1 & 3 & 3 \\ 4 & 6 & 2 & 3 & 1 & 1 & 6 & 7 & 3 & 4 & 3 & 2 & 2 & 8 & 4 & 4 & 2 & 3 \\ 2 & 4 & 0 & 1 & 0 & 0 & 3 & 3 & 0 & 2 & 1 & 0 & 0 & 4 & 4 & 2 & 1 & 1 \\ 3 & 4 & 1 & 1 & 0 & 0 & 4 & 4 & 2 & 2 & 1 & 1 & 1 & 4 & 2 & 4 & 1 & 1 \\ 2 & 2 & 1 & 3 & 1 & 1 & 2 & 1 & 1 & 2 & 4 & 3 & 3 & 2 & 1 & 1 & 5 & 4 \\ 2 & 2 & 1 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 6 & 5 & 3 & 3 & 1 & 1 & 4 & 8 \end{pmatrix}$$

$'_{ij}$ = # of events at which
woman i and woman j
jointly present if $i \neq j$.

$'_{ij}$ = total # of events
attended by woman i
if $i = j$.

Southern Women

Undirected, Weighted Graph

